Design and design check for multi layer well systems

MWell

Deltares systems

Deltares

Enabling Delta Life

Verification Report

MWELL

Design and design check for multi layer well systems

Verification Report

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Contents

List of Figures

List of Tables

Introduction

Deltares Systems commitment to quality control and quality assurance has led them to develop a formal and extensive procedure to verify the correct working of all of their geotechnical engineering tools. An extensive range of benchmark checks have been developed to check the correct functioning of each tool. During product development these checks are run on a regular basis to verify the improved product. These benchmark checks are provided in the following sections, to allow the user to overview the checking procedure and verify for themselves the correct functioning of MWELL.

The benchmarks are subdivided into five separate groups as described below.

- **Group 1 (chapter 1) - Benchmarks from literature (exact solution)**
  Simple benchmarks for which an exact analytical result is available from literature.
- **Group 2 (chapter 2) - Benchmarks from literature (approximate solution)**
  More complex benchmarks described in literature for which an approximate solution is known.
- **Group 3 (chapter 3) - Benchmarks from spread sheets**
  Benchmarks which test program features specific to MWELL.
- **Group 4 (chapter 4) - Benchmarks generated by MWELL**
  Benchmarks for which the reference results are generated using MWELL.
- **Group 5 (chapter 5) - Benchmarks compared with other programs**
  Benchmarks for which the results of MWELL are compared with the results of other programs.

The number of benchmarks in group 1 will probably remain the same in the future. The reason for this is that they are very simple, using only the most basic features of the program.

The benchmarks in group 2 are well documented in literature. There are no exact solutions available for these problems; however in the literature estimated results are available. When verifying the program, the results should be close to the results found in the literature.

The number of benchmarks in groups 3, 4 and 5 will grow as new versions of the program are released. These benchmarks are designed so that (new) features specific to the program can be verified. The benchmarks are kept as simple as possible so that only one specific feature is verified from one benchmark to the next.

As much as software developers would wish they could, it is impossible to prove the correctness of any non-trivial program. Re-calculating all the benchmarks in this report, and making sure the results are as they should be, proves to some degree that the program works as it should. Nevertheless, there will always be combinations of input values that will cause the program to crash or to produce wrong results. Hopefully by using the verification procedure the number of ways this can occur will be limited. The benchmarks are all described in sufficient detail for reproduction to be possible at any time. The information given is enough to be able to make the calculation. The input files can be found on CD-ROM or can be downloaded from our website www.deltaressystems.com.

1 Group 1: Benchmarks from literature (exact solution)

1.1 Huisman & Kemperman solution for a two layers system

1.1.1 Description

Formula Huisman & Kemperman (Huisman and Kemperman (1951)) is an analytical solution for a two layered system. Two benchmarks with the same geometry are made:

- Case I (bm1-1a): discharge Q_1 of 50 m^3/day from first aquifer;
- Case II (bm1-1b): discharge Q_2 of 30 m^3/day from second aquifer.

The same geometry is used for both benchmarks (see Figure 1.1) with:

Resistance	c_1	= 300 days
Transmissibility	$k_1 H_1$	= 10 m^2/day
Resistance	c_2	= 100 days
Transmissibility	$k_2 H_2$	= 20 m^2/day

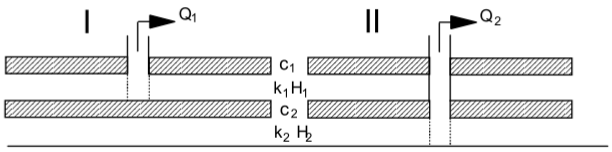


Figure 1.1: *Geometry of benchmark 1-1*

1.1.2 Benchmark results

According to Huisman & Kemperman (Huisman and Kemperman, 1951):

- For case I:
  - Drop of load in first aquifer [1]:
    $S_1 = \dfrac{Q_1}{2\pi\, k_1 H_1} \frac{1}{\lambda_1 - \lambda_2} \left[(\lambda_1 - \alpha_2)\ k_0 \left(\sqrt{\lambda_1} r\right) + (\alpha_2 - \lambda_2)\ k_0 \left(\sqrt{\lambda_2} r\right) \right]$
  - Drop of load in second aquifer [2]:
    $S_2 = \dfrac{Q_1}{2\pi\, k_1 H_1} \frac{\alpha_2}{\lambda_1 - \lambda_2} \left[-\ k_0 \left(\sqrt{\lambda_1} r\right) +\ k_0 \left(\sqrt{\lambda_2} r\right) \right]$
- For case II:
  - Drop of head in first aquifer [1]:
    $S_1 = \dfrac{Q_2}{2\pi\, k_2 H_2} \frac{\beta_1}{\lambda_1 - \lambda_2} \left[-\ k_0 \left(\sqrt{\lambda_1} r\right) +\ k_0 \left(\sqrt{\lambda_2} r\right) \right]$
  - Drop of head in second aquifer [2]:
    $S_2 = \dfrac{Q_2}{2\pi\, k_2 H_2} \frac{1}{\lambda_1 - \lambda_2} \left[(\alpha_2 - \lambda_2)\ k_0 \left(\sqrt{\lambda_1} r\right) + (\lambda_1 - \alpha_2)\ k_0 \left(\sqrt{\lambda_2} r\right) \right]$

where:

$$\alpha_1 = \frac{1}{k_1 H_1\, c_1} = 3.333 \times 10^{-4}$$

$$\alpha_2 = \frac{1}{k_2 H_2\, c_2} = 5 \times 10^{-4}$$

$$\beta_1 = \frac{1}{k_1 H_1\, c_2} = 10^{-3}$$

$$\lambda_1 = 0.5 \times \left(\alpha_1 + \alpha_2 + \beta_1 + \sqrt{(\alpha_1 + \alpha_2 + \beta_1)^2 - 4\,\alpha_1 \times \alpha_2} \right) = 1.737 \times 10^3$$

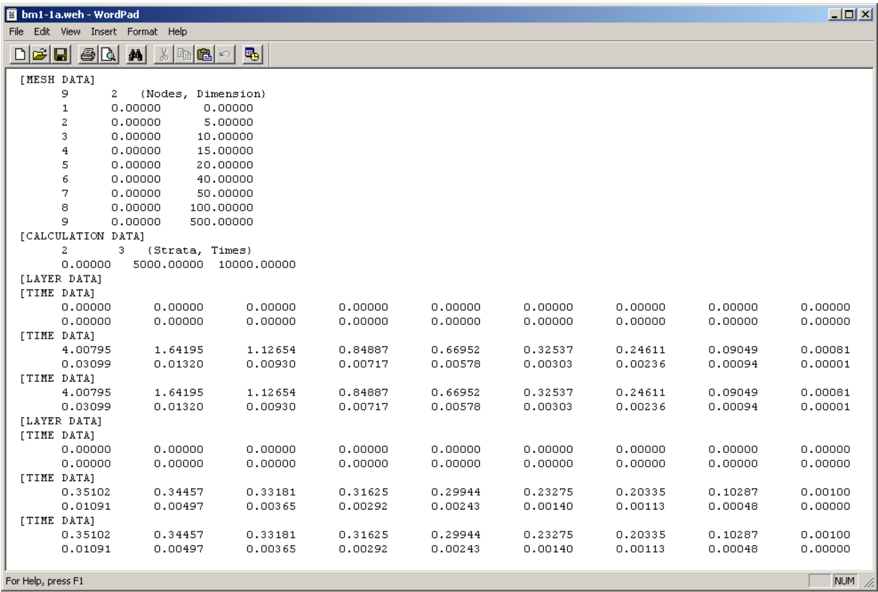
$$\lambda_2 = 0.5 \times \left(\alpha_1 + \alpha_2 + \beta_1 - \sqrt{(\alpha_1 + \alpha_2 + \beta_1)^2 - 4\,\alpha_1 \times \alpha_2} \right) = 9.593 \times 10^5$$

k_0 is the modified Bessel function of the second kind and order zero;
r is the distance to the well, in m.

The analytical results from this formula are performed in a spreadsheet for different distances to the well. The calculated results are given in Table 1.1 and Table 1.2.

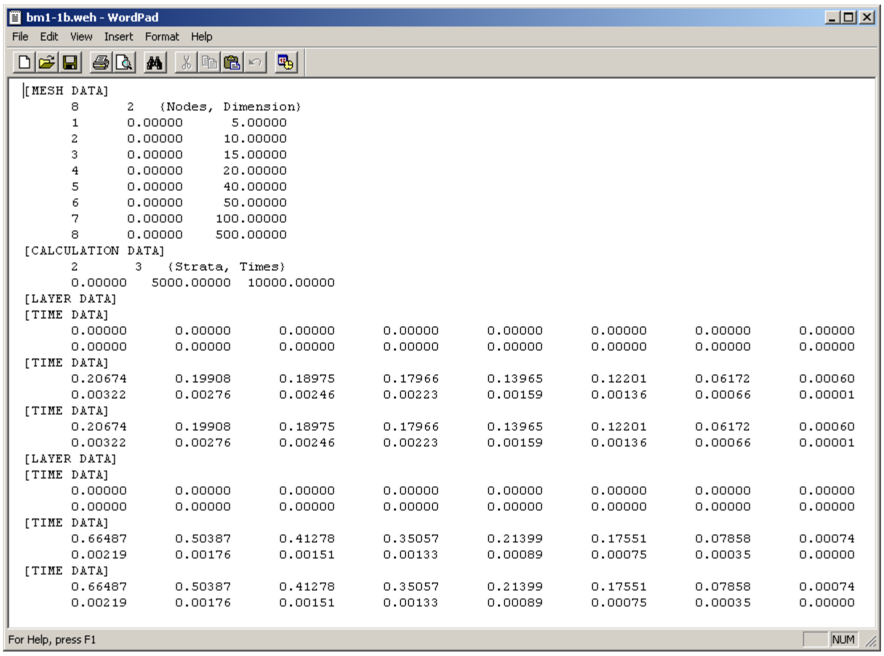
1.1.3 MWELL results

The numerical values of the drawdown in both aquifers can be found in the *.weh file as shown in Figure 1.2 and Figure 1.3. Comparison between MWell and Huisman & Kemperman solution is excellent.



```
[MESH DATA]
      9       2   {Nodes, Dimension}
      1       0.00000       0.00000
      2       0.00000       5.00000
      3       0.00000      10.00000
      4       0.00000      15.00000
      5       0.00000      20.00000
      6       0.00000      40.00000
      7       0.00000      50.00000
      8       0.00000     100.00000
      9       0.00000     500.00000
[CALCULATION DATA]
      2        3   {Strata, Times}
      0.00000   5000.00000  10000.00000
[LAYER DATA]
[TIME DATA]
      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
[TIME DATA]
      4.00795      1.64195      1.12654      0.84887      0.66952      0.32537      0.24611      0.09049      0.00081
      0.03099      0.01320      0.00930      0.00717      0.00578      0.00303      0.00236      0.00094      0.00001
[TIME DATA]
      4.00795      1.64195      1.12654      0.84887      0.66952      0.32537      0.24611      0.09049      0.00081
      0.03099      0.01320      0.00930      0.00717      0.00578      0.00303      0.00236      0.00094      0.00001
[LAYER DATA]
[TIME DATA]
      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000      0.00000
[TIME DATA]
      0.35102      0.34457      0.33181      0.31625      0.29944      0.23275      0.20335      0.10287      0.00100
      0.01091      0.00497      0.00365      0.00292      0.00243      0.00140      0.00113      0.00048      0.00000
[TIME DATA]
      0.35102      0.34457      0.33181      0.31625      0.29944      0.23275      0.20335      0.10287      0.00100
      0.01091      0.00497      0.00365      0.00292      0.00243      0.00140      0.00113      0.00048      0.00000
```


Figure 1.2: *MWell results for benchmark 1-1a*



```
bm1-1b.weh - WordPad
[MESH DATA]
      8      2   (Nodes, Dimension)
      1      0.00000      5.00000
      2      0.00000     10.00000
      3      0.00000     15.00000
      4      0.00000     20.00000
      5      0.00000     40.00000
      6      0.00000     50.00000
      7      0.00000    100.00000
      8      0.00000    500.00000
[CALCULATION DATA]
      2        3   (Strata, Times)
      0.00000   5000.00000  10000.00000
[LAYER DATA]
[TIME DATA]
      0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000
      0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000
[TIME DATA]
      0.20674     0.19908     0.18975     0.17966     0.13965     0.12201     0.06172     0.00060
      0.00322     0.00276     0.00246     0.00223     0.00159     0.00136     0.00066     0.00001
[TIME DATA]
      0.20674     0.19908     0.18975     0.17966     0.13965     0.12201     0.06172     0.00060
      0.00322     0.00276     0.00246     0.00223     0.00159     0.00136     0.00066     0.00001
[LAYER DATA]
[TIME DATA]
      0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000
      0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000     0.00000
[TIME DATA]
      0.66487     0.50387     0.41278     0.35057     0.21399     0.17551     0.07858     0.00074
      0.00219     0.00176     0.00151     0.00133     0.00089     0.00075     0.00035     0.00000
[TIME DATA]
      0.66487     0.50387     0.41278     0.35057     0.21399     0.17551     0.07858     0.00074
      0.00219     0.00176     0.00151     0.00133     0.00089     0.00075     0.00035     0.00000
For Help, press F1
```

Figure 1.3: *MWell results for benchmark 1-1b*

Table 1.1: *Results of benchmark 1-1a: Case I*

Distance	Drawdown in first aquifer			Drawdown in second aquifer		
	Benchmark	MWELL	Error	Benchmark	MWELL	Error
[m]	[m]	[m]	[%]	[m]	[m]	[%]
5	1.64195	1.64195	0.00	0.34457	0.34457	0.00
10	1.12654	1.12654	0.00	0.33181	0.33181	0.00
15	0.84887	0.84887	0.00	0.31625	0.31625	0.00
20	0.66952	0.66952	0.00	0.29944	0.29944	0.00
40	0.32537	0.32537	0.00	0.23275	0.23275	0.00
50	0.24611	0.24611	0.00	0.20335	0.20335	0.00
100	0.09049	0.09049	0.00	0.10287	0.10287	0.00
500	0.00081	0.00081	0.00	0.00100	0.00100	0.00

Table 1.2: *Results of benchmark 1-1b: Case II*

Distance	Drawdown in first aquifer			Drawdown in second aquifer		
	Benchmark	MWELL	Error	Benchmark	MWELL	Error
[m]	[m]	[m]	[%]	[m]	[m]	[%]
5	0.20674	0.20674	0.00	0.66487	0.66487	0.00
10	0.19908	0.19908	0.00	0.50387	0.50387	0.00
15	0.18975	0.18975	0.00	0.41278	0.41278	0.00
20	0.17966	0.17966	0.00	0.35057	0.35057	0.00
40	0.13965	0.13965	0.00	0.21399	0.21399	0.00
50	0.12201	0.12201	0.00	0.17551	0.17551	0.00
100	0.06172	0.06172	0.00	0.07858	0.07858	0.00
500	0.00060	0.00060	0.00	0.00074	0.00074	0.00

Use MWell input files bm1-1a.wei and bm1-1b.wei to run this benchmark.

1.2 Hantush & Jacob solution for leaky aquifers

1.2.1 Description

For wells in a leaky aquifer an approximate solution valid for small values of time is known under the name of Hantush & Jacob (Hantush and Jacob (1955)). The solution assumes no storage in the aquitard and also accounts for partially penetrating wells.

This benchmark considers two wells in a one layer system. A semi pervious layer with resistance c = 5000 days of an aquifer with a coefficient of transmissibility k_H = 200 m^2/day and a specific yield S = 25.03%. Both wells have the following characteristics:

- Well number 1:
  - X co-ordinate: 0 m
  - Z co-ordinate: 0 m
  - Discharge: Q_1 =100 m^3/day
- Well number 2:
  - X co-ordinate: 0 m
  - Z co-ordinate: 10 m
  - Discharge: Q_2 =50 m^3/day

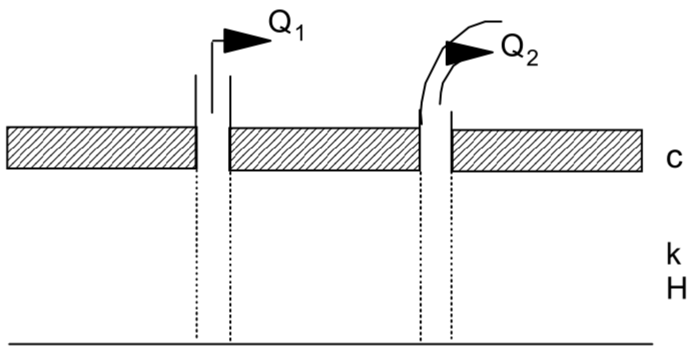


Figure 1.4: *Geometry of benchmark 1-2*

1.2.2 Benchmark results

According to Hantush & Jacob solution (Hantush and Jacob (1955)), the drop of head is given by the next formula:

$$\Delta h\left(r,t\right)=\frac{-Q}{4\pi\ kH}W\left(u,\frac{r}{B}\right) \tag{1.1}$$

where:
$u=\dfrac{S}{4kH}\dfrac{r^2}{t}$

r	is the distance to the well, in m;
t	is the time, in days;
B	is the leakage length, in m: $B=\sqrt{kH\cdot c}=\sqrt{200\times 5000}=1000\ \mathrm{m}$
W	is the logarithmic integral: $W\left(u,\frac{r}{B}\right)=\int\limits_u^\infty \frac{1}{y}\exp\left(-y-\frac{r^2}{4B^2y}\right)\,dy$

The analytical results from this formula are performed in a spreadsheet for different times and different distances to the wells. The calculated results are given in the tables below.

1.2.3 MWell results

The numerical values of the drawdown at different monitoring points and different times can be found in the *.weh file. Comparison between MWell and Hantush & Jacob solution is very good.

Table 1.3: *Results of benchmark 1-2 at time 1, 2 and 5 days*

		Benchmark			MWELL			Error		
Time [days]:		1	2	5	1	2	5	1	2	5
X [m]	Z [m]	[m]	[m]	[m]	[m]	[m]	[m]	[%]	[%]	[%]
0	0	0.594	0.635	0.690	0.594	0.635	0.689	0.00	0.00	0.15
0	1	0.360	0.401	0.456	0.360	0.401	0.455	0.00	0.00	0.22
0	2	0.310	0.351	0.405	0.309	0.350	0.405	0.32	0.29	0.00
0	3	0.283	0.324	0.378	0.282	0.323	0.378	0.35	0.31	0.00
0	4	0.266	0.307	0.361	0.266	0.307	0.361	0.00	0.00	0.00
0	5	0.255	0.296	0.351	0.255	0.296	0.350	0.00	0.00	0.29
0	6	0.250	0.291	0.345	0.250	0.290	0.345	0.00	0.34	0.00
0	7	0.249	0.290	0.344	0.249	0.290	0.344	0.00	0.00	0.00
0	8	0.255	0.296	0.350	0.254	0.295	0.349	0.39	0.34	0.29

Table 1.4: *Results of benchmark 1-2 at time 10, 20 and 50 days*

		Benchmark			MWELL			Error		
Time [days]:		10	20	50	10	20	50	10	20	50
X [m]	Z [m]	[m]	[m]	[m]	[m]	[m]	[m]	[%]	[%]	[%]
0	0	0.731	0.772	0.825	0.730	0.771	0.824	0.14	0.13	0.12
0	1	0.497	0.538	0.591	0.496	0.537	0.590	0.20	0.19	0.17
0	2	0.446	0.487	0.540	0.445	0.486	0.540	0.22	0.21	0.00
0	3	0.419	0.460	0.513	0.419	0.459	0.513	0.00	0.22	0.00
0	4	0.402	0.443	0.497	0.402	0.443	0.496	0.00	0.00	0.20
0	5	0.392	0.433	0.486	0.391	0.432	0.485	0.26	0.23	0.21
0	6	0.386	0.427	0.480	0.386	0.426	0.480	0.00	0.23	0.00
0	7	0.386	0.426	0.480	0.385	0.426	0.479	0.26	0.00	0.21
0	8	0.391	0.432	0.485	0.390	0.431	0.484	0.26	0.23	0.21

Use MWell input file bm1-2.wei to run this benchmark.

2 Group 2: Benchmarks from literature (approximate solution)

This chapter contains benchmarks described in literature, for which an approximate solution is known (group 2).

3 Group 3: Benchmarks from spreadsheets

This chapter contains benchmarks which test program features specific to MWELL using spreadsheets (group 3).

4 Group 4: Benchmarks generated by MWell

This chapter contains benchmarks for which the reference results are generated using MWELL.

5 Group 5: Benchmarks compared with other programs

This chapter contains benchmarks for which the results of MWELL are compared with the results of other programs.

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